CFD.

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1 Ans —

Applications of CFD in Engineering

Aerospace Applications (Research tools)

(i) Aerospace Applications

Aircraft Design, Rockets Nozzles, missiles wings (delta wing (flow visualization over delta wing) Aerofoil, intakes, Subsonic to Supersonic and hypersonic flow problems, Combustion at high speed flows (Scramjet engines)

· CFD is analogous of wind tunnel results obtained in laboratory. they both represents sets of data for given flow configurations at different ~~ma~~ mach numbers and Reynolds numbers.

(ii) Automobile: Applications: — Study of fluid flows over a car, (Drag force) Combustion and ~~intermixing of f~~ chamber phenomena

(iii) mechanical Applications → Gas turbines heat exchanger, Explosions

(iv) meteorological Applications → weather prediction.

(v) Environmental Applications → Air and water pollution.

(vi) Chemical Applications :— mixing of two chemicals separation.

2. Ans Taylor's series

$$f(x+\Delta x) = f(x) + \frac{\partial f}{\partial x}\Delta x + \frac{\partial^2 f}{\partial x^2}\frac{(\Delta x)^2}{2} + \cdots + \frac{\partial^n f}{\partial x^n}\frac{(\Delta x)^n}{n!} + \quad \text{—— (1)}$$

$$u_{i+1,j} = u_{i,j} + \left(\frac{\partial u}{\partial x}\right)_{i,j}\Delta x + \left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j}\frac{(\Delta x)^2}{2} + \left(\frac{\partial^3 u}{\partial x^3}\right)_{i,j}\frac{(\Delta x)^3}{3!} + \cdots \quad \text{— (2)}$$

from equation (1), We can write

finite differenale representation

$$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i+1,j} - u_{i,j}}{\Delta x} - \left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j}\frac{\Delta x}{2} - \left(\frac{\partial^3 u}{\partial x^3}\right)_{i,j}\frac{(\Delta x)^2}{3!}$$

Truncation error

—— (3)

$$\left(\frac{\partial u}{\partial x}\right)_{j,n} \simeq \frac{u_{j+1,n} - u_{j,n}}{\Delta x} \quad \text{--- (4)}$$

In equation (3), the lowest order term is the truncation error involves $\Delta x$ to first power, hence the equation (4) is called first order accurate.

$$\left(\frac{\partial u}{\partial x}\right)_{j,n} = \frac{u_{j+1,n} - u_{j,n}}{\Delta x} + O(\Delta x)$$

Backward difference

$$u_{j-1,n} = u_{j,n} + \frac{\partial u}{\partial x}_{j,n}(-\Delta x) + \left(\frac{\partial^2 u}{\partial x^2}\right)_{j,n}\frac{(-\Delta x)^2}{2} + \left(\frac{\partial^3 u}{\partial x^3}\right)_{j,n}\frac{(-\Delta x)^3}{3!} + \cdots$$

$$u_{j-1,n} = u_{j,n} - \left(\frac{\partial u}{\partial x}\right)_{j,n}\Delta x + \left(\frac{\partial^2 u}{\partial x^2}\right)\frac{(\Delta x)^2}{2!} - \left(\frac{\partial^3 u}{\partial x^3}\right)_{j,n}\frac{(\Delta x)^3}{3!} \quad \text{--- (5)}$$

$$\left(\frac{\partial u}{\partial x}\right)_{j,n} = \frac{u_{j,n} - u_{j-1,n}}{\Delta x} + O(\Delta x)$$

(3) Central difference: —
write equation (1), (2) and (5), again.
equation (1) − equation (5), we get

$$u_{j+1,n} - u_{j-1,n} = 2\left(\frac{\partial u}{\partial x}\right)_{j,n} \Delta x + 2\left(\frac{\partial^3 u}{\partial x^3}\right)_{j,n} \frac{(\Delta x)^3}{3!} + \cdots$$

$$\left(\frac{\partial u}{\partial x}\right)_{j,n} = \frac{u_{j+1,n} - u_{j-1,n}}{2\Delta x} + O(\Delta x)^2$$

lowest order term involves $(\Delta x)^2$ which is second order accurate.

A. Laplace's equation

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \quad \text{—— (1)}$$

$$\varphi_{xx} + \varphi_{yy} = 0 \quad \text{—— (2)}$$

Original form of equation

$$A\varphi_{xx} + B\varphi_{xy} + C\varphi_{yy} + D\varphi_x + E\varphi_y + F\varphi + G = 0 \quad \text{—— (3)}$$

Where A, B, C are in function of (x, y).

~~Equating~~

Comparing equation (2) and (3), we get

$A = 1, \; B = 0, \; C = 1$

$\Delta = B^2 - 4AC$

$= 0 - 4 \times 1 \times 1$

$= -4$

$\Delta < 0$ This equation is elliptical in nature.

(5)

$$\frac{\partial^2 \varphi}{\partial t^2} = c^2 \frac{\partial^2 \varphi}{\partial x^2} \quad \text{—— (1)}$$

Let $x = X$

$t = Y$

$$\frac{\partial^2 \varphi}{\partial Y^2} = c^2 \frac{\partial^2 \varphi}{\partial X^2} \quad \text{——} \cancel{(2)}$$

$$c^2 \frac{\partial^2 \varphi}{\partial X^2} - \frac{\partial^2 \varphi}{\partial Y^2} = 0 \quad \text{——} \cancel{(2)}$$

$$c^2 \varphi_{XX} - \varphi_{YY} = 0 \quad \text{—— (2)}$$

original form of pde

$$A \varphi_{XX} + B \varphi_{XY} + C \varphi_{XX} + D \varphi_X + E \varphi_Y + F\varphi + G = 0 \quad \text{—— (3)}$$

Comparing equation (2) and (3), we get

$$A = c^2, \; B = 0, \; C = -1$$

$$D = B^2 - 4AC$$

$$= 0 - 4 \times c^2 \times -1 = 4c^2$$

$D > 0$ Hyperbolic in nature.

$$\frac{dy}{\partial x} = \frac{-B \pm \sqrt{D}}{2A}$$

$$= \frac{0 \pm \sqrt{4c^2}}{2c^2} = \pm \frac{1}{c}$$

$$\frac{dy}{\partial x} = \pm \frac{1}{c}$$

Substituting in reverse way we get

$$\frac{\partial t}{\partial x} = \pm \frac{1}{c}$$

$$\frac{\partial x}{\partial t} = \pm c$$

$$x = \pm ct + C$$

$$\left.\begin{aligned} &= x + ct \\ &\quad x - ct \end{aligned}\right\}$$

6. Ans : —

Write equation ①, ② and ⑤ from ~~question~~ answer ②.

Adding equation ① and ⑤, we get

$$u_{j+1,n} + u_{j-1,n} = 2u_{j,n} + \left(\frac{\partial^2 u}{\partial x^2}\right)_{j,n} (\Delta x)^2 + \left(\frac{\partial^4 u}{\partial x^4}\right)_{j,n} \frac{(\Delta x)^4}{12} + \cdots$$

solving for $\frac{\partial^2 u}{\partial x^2}$, we get

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{j,n} = \frac{u_{j+1,n} - 2u_{j,n} + u_{j-1,n}}{(\Delta x)^2} + O(\Delta x)^2$$

Classification of pde's.

$$A\varphi_{xx} + B\varphi_{xy} + C\varphi_{yy} + D\varphi_x + E\varphi_y + F\varphi + G = 0 \quad \text{--- (1)}$$

$$\varphi_{xx} = \frac{\partial^2\varphi}{\partial x^2}, \quad \varphi_{xy} = \frac{\partial^2\varphi}{\partial x\,\partial y}, \quad \varphi_{yy} = \frac{\partial^2\varphi}{\partial y^2}$$

$$\varphi_x = \frac{\partial\varphi}{\partial x}, \quad \varphi_y = \frac{\partial\varphi}{\partial y}, \quad \varphi(x,y)$$ ~~where $\varphi$ is in function of $(x,y)$~~

A, B, C are in function of $(x,y)$ only

characteristics of ~~classification of~~ pde's

$$\varphi_x(x,y), \quad d\varphi_x = \frac{\partial\varphi_x}{\partial x}dx + \frac{\partial\varphi_x}{\partial y}dy$$

$$= \varphi_{xx}dx + \varphi_{xy}dy \quad \text{--- (2)}$$

$$d\varphi_y = \frac{\partial\varphi_y}{\partial x}dx + \frac{\partial\varphi_y}{\partial y}dy$$

$\phi_{xy}\, dx + \phi_{yy}\, dy$ — (3)

from equation (1), (2), (3), we get

$$\begin{bmatrix} A & B & C \\ dx & dy & 0 \\ 0 & dx & dy \end{bmatrix} \begin{bmatrix} \phi_{xx} \\ \phi_{xy} \\ \phi_{yy} \end{bmatrix} = \begin{bmatrix} -H \\ d\phi_x \\ d\phi_y \end{bmatrix}$$

for $\phi_{xx}, \phi_{xy}, \phi_{yy}$, to be discontineous $\Delta = 0$

$$A(dy)^2 - B\,dy\,dx + C(dx)^2 = 0$$

$$A\left(\frac{dy}{dx}\right)^2 + C - B\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \cancel{-B \pm \sqrt{B^2 - 4A}}$$

If $B^2 - 4AC = 0$ it has one real characteristic (Parabola)

If $B^2 - 4AC < 0$ it has zero real characteristics.

If $B^2 - 4AC = 0$ it has two real characte